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On Regular Pre-Semiclosed Sets in Topological Spaces

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Abstract

The generalized closed sets in point set topology have been found considerable interest among general topologists. Veerakumar introduced and investigated pre-semi- closed sets and Anitha introduced pgpr-closed sets. In this article the concept of regular pre-semiclosed sets is introduced in topological spaces and its relationships with other generalized sets are investigated .

Keywords: pre-semiclosed, pgpr-closed, semi-preclosure, rg-open and g-open sets.

Introduction

Levine[9] introduced generalized closed (briefly g-closed) sets in topology. Researchers in topology studied several versions of generalized closed sets. In this paper the concept of regular pre-semiclosed (briefly rps-closed) set is introduced and their properties are investigated. This class of sets is properly placed between the class of semi-preclosed sets and the class of pre-semiclosed sets. Certain preliminary concepts are given in the section 2, the concept of rps-closedness is studied in section 3 and the reference is given at the end followed by a diagram that gives the relationships among the generalized closed sets in topological spaces.

Preliminaries

Throughout this paper X and Y represent the topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a topological space X, *cl*A and *int*A denote the closure of A and the interior of A respectively. X\A denotes the complement of A in X. Throughout the paper □ indicates the end of the proof. We recall the following definitions.

Definition 2.1

A subset A of a space X is called

(i) pre-open [12] if $A \subseteq$ *int cl*A and pre-closed if *cl int*A $\subseteq$ A;

(ii) semi-open [8] if $A \subseteq$ *cl int*A and semi-closed if *int cl*A $\subseteq$ A;

(iii) semi-pre-open [1] if $A \subseteq$ *cl int cl*A and semi-pre-closed if *int cl int*A $\subseteq$ A;

(iv) α-open [14] if $A \subseteq$ *int cl int*A and α-closed if *cl int clA* $\subseteq$ A;

(v) regular open [17] if A = *int cl*A and regular closed if A = *cl int*A.

(vi) л-open [22] if A is a finite union of regular open sets.

The semi-pre-closure(resp. semi-closure, resp. pre-closure, resp. α-closure) of a subset A of X is the intersection of all semi-pre-closed (resp. semi-closed, resp. pre-closed, resp. α-closed) sets containing A and is denoted by *spcl*A (resp. *scl*A, resp.*pcl*A, resp. α*cl*A).

Definition 2.2

A subset A of a space X is called

(i) generalized closed[9] (briefly g-closed) if *cl*A $\subseteq$U whenever A$\subseteq$U and U is open.

(ii) regular generalized closed[15](briefly rg-closed) if *cl*A $\subseteq$ U whenever A$\subseteq$U and U is regular open.

(iii) α-generalized closed[10](briefly αg-closed) if α*cl*A $\subseteq$ U whenever A$\subseteq$U and U is open.

(iv) generalized-semi pre-regular-closed [16] (briefly gspr-closed) if *spcl*A $\subseteq$ U whenever A$\subseteq$ U and U is regular-open.

(v) generalized semi-closed [3](briefly gs-closed) if *scl*A $\subseteq$ U whenever A$\subseteq$U and U is open.

(vi) л-generalized closed [5](briefly лg-closed) if *cl*A $\subseteq$U whenever A$\subseteq$U and U is *л*-open.

(vii) generalized pre-closed [11](briefly gp-closed) if *pcl*A$\subseteq$U whenever A$\subseteq$U and U is open.

(viii) generalized semi-pre-closed [4](briefly gsp-closed) if *spcl*A$\subseteq$U whenever A$\subseteq$U and U is open.

(ix) π-generalized pre-closed [7] (briefly πgp-closed) if *pcl*A$\subseteq$U whenever A$\subseteq$U and U is π-open.

(x) generalized pre-regular closed[6](briefly gpr-closed) if *pcl*A $\subseteq$U whenever A$\subseteq$U and U is regular open.

(xi) weakly generalized closed[13] (briefly wg-closed)if *cl int*A$\subseteq$ U whenever A$\subseteq$U and U is open.

(xii) π-generalized semi-pre-closed[16](briefly πgsp-closed) if *spcl*A $\subseteq$ U whenever A$\subseteq$U and U is π-open.

(xiii) regular weakly generalized closed[19](briefly rwg-closed) if *cl int*A$\subseteq$U whenever A$\subseteq$U and U is regular open.

The complements of the above mentioned closed sets are their respective open sets. For example a subset B of a space X is generalized open (briefly g-open) if X\B is g-closed.

Definition 2.3

A subset A of a space X is called

(i) pre-semiclosed [20] if *spcl*A$\subseteq$U whenever A$\subseteq$U and U is g-open.

(ii) pre-generalized pre-regular-closed[2] (briefly pgpr-closed) if *pcl*A$\subseteq$U whenever A$\subseteq$U and U is rg-open.

The complements of the above mentioned closed sets are their respective open sets.

The following lemmas will be useful in sequel.

Lemma 2.4 [2]

If A is semi closed then *pcl* (A$\cup$B) = *pcl* A $\cup$ *pcl* B.

Lemma 2.5 [1]

For any subset A of X, the following results hold:

(i) *scl*A = A$\cup$ *int cl*A ;

(ii) *pcl*A = A$\cup$ *cl int*A;

(iii) *spcl*A = A$\cup$ *int cl int*A.

Lemma 2.6 [18]

If A is semi closed in X , then *cl int*(A$\cup$B) = *cl int*A $\cup$ *cl int*B.

Lemma 2.7 [6]

If A is regular-open and gpr-closed then A is pre-closed and hence clopen.

Definition 2.8

A space X is called extremally disconnected [21] if the closure of each open subset of X is open

Regular pre-semiclosed sets

Veerakumar[20] introduced pre-semiclosed sets in the year 2002 and Anitha et al.[2] introduced pgpr-closed sets by replacing "*spcl*" by "*pcl*" and "g-open" by "rg-open" in the definition of pre-semiclosed sets. In an analog way the regular pre-semiclosed sets are defined by replacing "g-open" by "rg-open". If every rg-open neighbourhood of A contains its semipreclosure, then A is called a regular pre-semiclosed subset. The formal definition of this concept is as follows.

Definition 3.1

A subset A of a space X is called regular pre-semiclosed (briefly rps-closed) if $spcl A \subseteq U$ whenever $A \subseteq U$ and U is rg-open.

Proposition 3.2

(i) Every semi-pre-closed set is rps-closed.

(ii) Every pgpr-closed set is rps-closed.

(iii) Every pre-closed set is rps-closed.

(iv) Every α-closed set is rps-closed.

(v) Every regular closed set is rps-closed.

Proof

(i) Let A be a semi-pre-closed set in X. Since $spcl A = A$, it follows that A is rps-closed.

(ii) Let A be a pgpr-closed set in X. Let $A \subseteq U$ and U is rg-open. Since A is pgpr-closed,

$pcl A \subseteq U$. Again since $spcl\, A \subseteq pcl A$, we see that $spcl A \subseteq U$. Therefore A is rps-closed

(iii) follows from (ii) and the fact that every pre-closed set is pgpr-closed .

(iv) follows from (iii) and the fact that every α-closed set is pre-closed.

(v) Let A be a regular closed subset of X .Let $A \subseteq U$ and U is rg-open. Since A is regular-closed ,$A = cl\, int A$. $cl\, int A \subseteq U$ and U is rg-open. $int\, cl\, int A \subseteq int U \subseteq U$ and U is rg-open.

$A \cup int\, cl\, int A \subseteq A \cup U \subseteq U$ and U is rg-open. $spcl A \subseteq U$ whenever $A \subseteq U$ and U is rg-open. Therefore A is rps-closed.

□

The reverse implications are not true as shown in Example 3.4 and Example 3.5.

Proposition 3.3

(i) Every rps-closed set is pre-semi-closed.

(ii) Every rps-closed set is gspr-closed.

(iii) Every rps-closed set is gsp-closed.

Proof

(i) Let A be a rps-closed subset of a space X. Let A $\subseteq$ U where U is g-open. Since every g-open set is rg-open and since A is rps-closed , by Definition 2.3(i), A is pre-semi-closed.

(ii) Let A be a rps-closed subset of a space X . Let A $\subseteq$ U and U is regular-open. Since every regular-open set is rg-open and since A is rps-closed, by Definition 2.2 (iv), *spcl*A $\subseteq$ U.

Therefore A is gspr-closed.

(iii) Let A be a rps-closed subset of a space X. Let A $\subseteq$ U and U is open . Since every open set is g-open and since every g-open set is rg-open, *spclA* $\subseteq$ U and hence A is gsp-closed.

□

The reverse implications are not true as shown in Example 3.4.

Example 3.4

Let X = {a,b,c,d} with topology τ = {φ,{a},{b},{a,b},{b,c},{a,b,c},X}. Then

(i) {a,b,d} is rps-closed but not semi-pre-closed.

(ii) {b,d} is pre-semiclosed but not rps-closed .

(iii) {a} is rps-closed but not pgpr-closed set.

(iv) {a,b} is gspr-closed but not rps-closed.

(v) {b,c} is rps-closed but not pre-closed.

(vi) {b,c} is rps-closed but not α-closed.

(vii){b,d} is gsp -closed but not rps-closed.

Example 3.5

Let X = {a,b,c} with topology τ = {φ,{a,b},X}. Then {a} is rps-closed but not regular-closed.

The concept of rwg-closed, wg-closed, gpr-closed, πg-closed, πgp-closed, gp-closed, rg-closed, αg-closed sets are independent with the concept of rps-closed as shown in the following example.

Example 3.6

Let X = {a,b,c,d} with topology τ = {φ,{a},{b},{a,b},{b,c},{a,b,c},X}.

(i) {a} is rps-closed but not rwg-closed and {a,b} is rwg-closed but not rps-closed.

(ii) {a} is rps-closed but not wg-closed and {b,d} is wg-closed but not rps-closed.

(iii) {a} is rps-closed but not gpr-closed and{b} is gpr-closed but not rps-closed.

(iv) {a} is rps-closed but not πg-closed and {b,d} is πg-closed but not rps-closed.

(v) {a} is rps-closed but not πgp-closed and {b,d} is πgp-closed but not rps-closed

(vi) {b,d} is gp-closed but not rps-closed and {a,b,d} is rps-closed but not gp-closed.

(vii) {a,b} is rg-closed but not rps-closed and {a} is rps-closed but not rg-closed.

(viii) {a} is rps-closed but not αg-closed and {b,d} is αg-closed but not rps-closed.

The concept of g-closed and rps-closed sets are independent as shown in the following example.

Example 3.7

Let X = {a,b,c,d} with $\tau = \{\varphi,\{a\},\{a,b\},X\}$. Then {b} is rps-closed but not g-closed and {a,c} is g-closed but not rps-closed.

The concept of gs-closed and rps-closed sets are independent as shown in the following example.

Example 3.8

Let X = {a,b,c} with topology $\tau = \{\varphi,\{a,b\},X\}$. Then {a} is rps-closed but not gs-closed.

From Example 3.6 we see that {b,d} is gs-closed but not rps-closed.

Thus the above discussions lead to the implication diagram given at the end. In this diagram by "A $\longrightarrow$ B" we mean A implies B but not conversely and

"A $\not\leftrightarrow$ B" means A and B are independent of each other.

The Union and intersection of two rps-closed sets need not be rps-closed as shown in the following example.

Example 3.9

Let X = {a,b,c,d} with $\tau = \{\varphi,\{a\},\{b\},\{a,b\},\{b.c\},\{a,b,c\},X\}$. Then A = {a} , B = {b,c} and C = {a,b,d}.Here A and B are rps-closed but $A \cup B$ = {a,b,c} is not rps-closed . Also B and C are rps-closed but $B \cap C$ = {b} is not rps-closed .

Theorem 3.10

If A is regular-open and A is gpr-closed then A is (i) rps-closed (ii) gspr-closed.

Proof

Follows from Lemma 2.7 and Diagram 1

□

Theorem 3.11

If A is semi-closed then *spcl* (A∪B) = *spcl*A∪ *spcl*B.

Proof

Suppose A is semi-closed. By Lemma 2.5(iii),

spcl (A∪B) = (A∪B) ∪ *int cl int*(A∪B).

spcl(A∪B) = (A∪B) ∪ *int* [*cl int*A∪*cl int*B] by applying Lemma 2.6.

= (A∪B) ∪ [*int cl int*A∪*int cl int*B]

=(A∪ *int cl int*A] ∪ [B∪ *int cl int*B]

= [A∪ *int cl int*A] ∪ [B∪ *int cl int*B]

spcl (A∪B) = *spcl*A ∪ *spcl*B.

□

Theorem 3.12

Let A and B be rps-closed sets and let A be semi-closed. Then $A \cup B$ is rps-closed.

Proof

Let $A \cup B \subseteq U$ and U be rg-open. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are rps-closed sets $spclA \subseteq U$ and $spclB \subseteq U$. Therefore $spclA \cup spclB \subseteq U$. Since A is semi-closed, by Theorem 3.11, $spcl(A \cup B) \subseteq U$. Hence $A \cup B$ is rps-closed.

□

Theorem 3.13

If a set A is rps-closed then, $spclA \setminus A$ does not contain a non empty rg-closed set.

Proof

Suppose that A is rps-closed. Let F be a rg-closed subset of $spclA \setminus A$. Then $F \subseteq spclA \cap (X \setminus A) \subseteq X \setminus A$ and so $A \subseteq X \setminus F$. But A is rps-closed. Since $X \setminus F$ is rg-open, $spclA \subseteq X \setminus F$ that implies $F \subseteq X \setminus spclA$. As we have already $F \subseteq spclA$, it follows that $F \subseteq spclA \cap (X \setminus spclA) = \emptyset$. Thus $F = \emptyset$. Therefore $spclA \setminus A$ does not contain a non empty rg-closed set .

□

Theorem 3.14

Let A be rps-closed. Then A is semi-pre-closed if and only if $spclA \setminus A$ is rg-closed.

Proof

If A is semi-pre-closed then $spcl(A) = A$ and so $spclA \setminus A = \emptyset$ which is rg-closed.
Conversely, suppose that $spclA \setminus A$ is rg-closed. Since A is rps-closed,
by Theorem 3.13, $spclA \setminus A = \emptyset$. That is $spclA = A$ and hence A is semi-pre-closed.

□

Theorem 3.15

If A is rps-closed and if $A \subseteq B \subseteq spclA$ then

(i) B is rps-closed

(ii) $spclB \setminus B$ contains no non empty rg-closed set.

Proof

$A \subseteq B \subseteq spclA \Rightarrow spclB = spclA$. Now suppose $B \subseteq U$ and U is rg-open. Since A is rps-closed and since $A \subseteq B \subseteq U$, $spclA \subseteq U$ that implies $spclB \subseteq U$. This proves (i). Since B is rps-closed, (ii) follows from Theorem 3.13.

□

Theorem 3.16

For every point x of a space X, $X \setminus \{x\}$ is rps-closed or rg-open.

Proof

Suppose $X \setminus \{x\}$ is not rg-open .Then X is the only rg-open set containing $X \setminus \{x\}$. This implies $spcl(X \setminus \{x\}) \subseteq X$. Hence $X \setminus \{x\}$ is rps-closed set in X. □

Theorem 3.17

Suppose A is rg-open and A is rps-closed. Then A is semi-pre-closed.

Proof

Since A is rg-open and since A is rps-closed ,A $\subseteq$ A $\Rightarrow$ *spcl*A $\subseteq$ A. This proves the theorem.

□

Theorem 3.18

Let A be rps-closed and *cl int* A be open. Then A is pgpr-closed.

Proof

Let A $\subseteq$ U and U be rg-open. Since A is rps-closed, *spcl*A $\subseteq$ U . By Lemma 2.5 (iii)

A $\cup$ *int cl int*A $\subseteq$ U that implies A $\cup$ *cl int*A $\subseteq$ U . Applying Lemma 2.5 (ii) *pcl*A $\subseteq$ U.

Therefore A is pgpr-closed.

□

Corollary 3.19

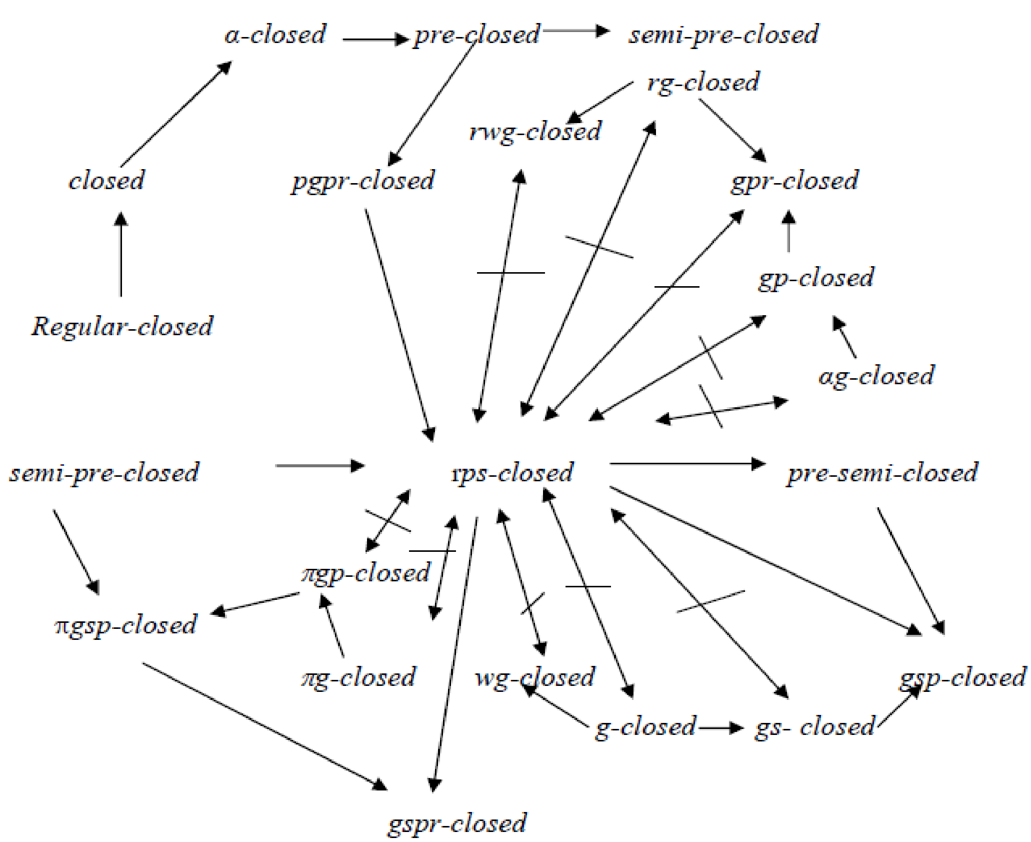
In an extremally disconnected space X, every rps-closed set is pgpr-closed.

Proof

In an extremally disconnected space X, *cl int*A is open for every subset A of X. Then the Corollary follows directly from Theorem 3.18.

□

Diagram 1



References

[1] Andrijevic D (1986) Semi-preopen sets. Mat Vesnik 38: 24-32.

[2] Anitha M and P Thangavelu (2005) On Pre-Generalized Pre-Regular-Closed sets. *Acta Ciencia Indica* 31M (4): 1035-1040.

[3] Arya S and T Nour(1990) Characterizations of s-normal spaces. Indian J. Pure Appl. Math 21: 717-719.

[4] Dontchev J (1995) On generalizing semi-pre-open sets. Mem Fac Sci Kochi Univ Ser A Math. 16: 35-48.

[5] Dontchev J and T Noiri (2000) Quasi-normal spaces and Πg-closed sets. Acta Math. Hungar 89 (3): 211-219.

[6] Gnanambal Y(1997) On generalized pre regular closed sets in topological spaces. *Indian J. Pure Appl. Math.* 28 (3): 351-360.

[7] Janaki C (1999) Studies on $\pi g\alpha$-closed sets in topology. Ph.d Thesis. Bharathiar university Coimbatore, India.

[8] Levine N (1963) Semi-open sets and semi continuity in topological spaces. Amer. Math. Monthly 70: 36-41.

[9] Levine N (1970) Generalized closed sets in topology. Rend. Circ. Mat. Palermo 19 (2): 89-96.

[10] Maki H, R Devi and K Balachandran (1994) Associated topologies of generalized α -closed Sets and α -generalized closed sets. Mem. Fac. Sci. Kochi Univ. (Math) 15: 51-63.

[11] Maki H, J Umehara and T Noiri (1996) Every topological space is pre-$T_{1/2}$. Mem Fac Sci Kochi Univ Ser.A, Math17: 33-42.

[12] Mashhour AS, ME Abd El-Monsef and SN El-Deeb (1982) On precontinuous and weak Precontinuous functions. Proc. Math. Phys. Soc . Egypt 53: 47-53.

[13] Nagaveni N (1999) Studies on generalizations of homeomorphisms in topological spaces. *Ph.D Thesis*. Bharathiar University, Coimbatore, India.

[14] Njastad O (1965) On some classes of nearly open sets. Pacific J. Math.15: 961-970.

[15] Palaniappan N and KC Rao (1993) Regular generalized closed sets. Kyungpook Math. J 33: 211-219.

[16] Sarsak MS and N Rajesh (2010) π-Generalized semi-pre-closed sets. International Mathematical Forum 5(12): 573-578.

[17] Stone M (1937) Application of the theory of Boolean rings to general topology. *Trans. Amer. Math. Soc* 41: 374-481.

[18] Thangavelu P and KC Rao (2002) p-sets in topological spaces. Bulletin of Pure and Applied Sciences 21E (2): 341-355.

[19] Vadivel A and K Vairamanickam (2009) rgα-closed sets and rgα-open sets in topological Spaces. Int J Math Analysis 3 (37): 1803-1819.

[20] Veerakumar MKRS (2002) Pre-semi-closed sets. Indian J. Math 44 (2): 165-181.

[21] Willard S (1970) General Topology. Addison Wesley.

[22] Zaitsav V(1968) On certain classes of topological spaces and their bicompactifications. Dokl. Akad. Nauk SSSR 178: 778-779.